Evolution of Acoustic Logic Gates in Granular Metamaterials

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Abstract. Granular metamaterials are a promising choice for the realization of mechanical computing devices. As preliminary evidence of this, we demonstrate here how to embed Boolean logic gates (AND and XOR) into a granular metamaterial by evolving where particular grains are placed in the material. Our results confirm the existence of gradients of increasing "AND-ness" and "XOR-ness" within the space of possible materials that can be followed by evolutionary search. We measure the computational functionality of a material by probing how it transforms bits encoded as vibrations with zero or non-zero amplitude. We compared the evolution of materials built from mass-contrasting particles and materials built from stiffness-contrasting particles, and found that the latter were more evolvable. We believe this work may pave the way toward evolutionary design of increasingly sophisticated, programmable, and computationally dense metamaterials with certain advantages over more traditional computational substrates.

Keywords: Granular metamaterials · Mechanical computing · Inverse design problem

1 Introduction

The concept of mechanical computing can be traced back to the second century BC when the earliest known analogue computer, the Antikythera mechanism, was invented [4]. Since then many other mechanical devices were invented for applications other than astronomical calculations such as basic mathematical operations [18], solving arbitrary equations [5], and even differentiation and

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integration [21]. With the invention of operational amplifiers in the early twentieth century, electronic analog computers became feasible. Soon after, digital computers emerged and rapidly became the dominant form for computation [9]. Although they impose a more abstract form of system representation, digital computers rapidly outpaced their mechanical counterparts. Moreover, higher precision and the capacity for miniaturization helped make the digital computing paradigm more desirable. But, recently, rapid advances in the chemical, biological and materials sciences have opened new opportunities for embedding computation directly into physical substrates [20].

Metamaterials are one such promising class of substrate that has surfaced in recent years. Metamaterials are engineered composite materials that exhibit properties different from their constituent materials, and from material properties observed in nature [7]. Granular metamaterials (GMMs) are a specific class of metamaterials consisting of discrete particles. GMMs exhibit increased plasticity compared to continuous metamaterials because they can be dynamically programmed by reconfiguring the material's physical structure or changing particles' properties using external stimuli [19]. In this paper, we investigate the potential of granular metamaterials as a physical substrate for mechanical computation.

Starting with logic gates as the basic computational blocks upon which more complex units can be built, we focus our work on evolving GMMs that act as acoustic logic gates: vibrations with near-zero and non-zero amplitudes arriving at and leaving the material are treated as incoming and outgoing bits. There are several advantages for this type of computation compared to the conventional approach of designing logic gates using electrical transistors. First, by moving to a mechanical substrate, we can avoid analogue to digital conversion, also thereby bypassing all of the limitations of abstract representations and discretizations necessary for a digital computing system [21]. Second, outsourcing computation to the physical substrate provides opportunities for conflating computational, mechanical, energetic, sensing and actuation properties into the same material. This could lead to robots built from continua of materials rather than modular components. This in turn could allow these machines to better exploit the natural dynamics of the materials, leading to better energy efficiency and higher robustness and stability. Finally, our approach affords a bottom-up design view point for computer architecture where the exact form of computation is not predetermined [11] and useful, non-intuitive exploitations of the material itself can be found by evolutionary search [16].

Some work on embedding mechanical computation into materials has been conducted. In [15], a universal logic gate is implemented as a nonlinear mass-spring-damper model. In [13] a soft bistable building block is designed and used in the implementation of soft mechanical diodes and logic gates. [6] utilizes a bistable spring embedded in a unit cell to implement simple logic gates. In [17] connected origami units are used to program the behavior of a mechanical bit and thus produce logic gates. [8] and [1] present examples of acoustic gate design in a 1D chain of elastic particles. Despite these advances, in none of the aforementioned works is the material automatically optimized for the desired computational function. Instead, the building blocks are hand-designed based on human intuition. In contrast, we here propose using evolutionary algorithms for automatically optimizing materials to exhibit desired computation. Moreover, the abovementioned works involved continuous metamaterials or 1D particle chains, while we here investigate the computational potential of 2D granular metamaterials. This type of material can exhibit different responses to different environmental stimuli by reconfiguring their physical structure and changing the material properties of individual particles. Thus, we anticipate a greater potential for extending our work to more complex computational operations.

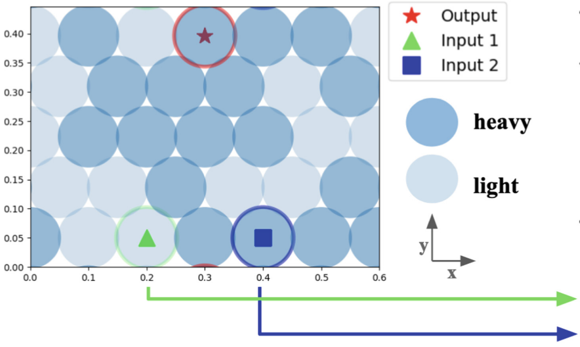
Granular metamaterials have many parameters that affect the response of the system. For example, the position, size, shape, stiffness and mass of each particle can affect the eigenfrequencies of the system and consequently the propagation of acoustic waves through the material. With so many design parameters, deciding on the optimal micro-structure to achieve a desired macro-behavior (i.e. a logic gate) is a non-trivial optimization problem. For this reason, evolutionary algorithms have already proven useful for designing metamaterials that exhibit mechanical properties [10,12] rather than the computational properties we study here. In this paper, we apply evolutionary algorithms to the design of 2D granular metamaterials to act as acoustic logic gates, where the input and output signals are acoustic waves. The granular assembly is designed such that it passes or filters the propagation of certain waves and thus acts as a Boolean logic gate.

The remainder of the paper is organized as follows: first, we formally define the problem and introduce the simulator used for optimization. Then, simulation results are presented for two different cases: designing an AND gate and designing an XOR gate. Finally, the results are discussed and the paper is concluded with some future possible directions of work.

2 Problem Statement

Our material is comprised of a two dimensional assembly of two types of circular particles placed on a hexagonal lattice. As mentioned in the introduction, the goal is to reconfigure this granular material to act as a logic gate where the inputs and the output receive and emit acoustic waves respectively. The setup is shown in Fig. 1. We choose two particles on one side of the material to serve as the input ports and one particle on the other side to serve as the output port. Each input particle i receives a sinusoidal wave with amplitude A_i and frequency ω applied in the x direction (the particle is vibrated left and right). When this signal is applied to a particle, it causes a displacement from its initial position x_i^0.

In order to represent a logic gate in this substrate, a representation for the bits must be chosen. One option is to use the amplitude of the displacement signal $(x_i(t))$ as the bit abstraction. In this case, applying a sinusoidal wave with amplitude zero $(A_i = 0)$ to an input port denotes the presentation of a zero at that port. We fixed the non-zero amplitude to 1×10^{-2}, which is 10%



A_1	A_2	*output*
0	0	O_{00}
0	$0.1D$	O_{01}
$0.1D$	0	O_{10}
$0.1D$	$0.1D$	O_{11}

$$x_1(t) = x_1^0 + A_1 \sin(\omega t)$$

$$x_2(t) = x_2^0 + A_2 \sin(\omega t)$$

Fig. 1. Problem formulation: a logic gate with two inputs (green and blue) and one output (red) embedded in a 2D granular assembly composed of heavy (dark blue) and light (light blue) particles. A_1 and A_2 denote the amplitudes of oscillations applied in the x direction to the input ports (D = particle diameter). ω denotes the input frequency. The truth table indicates how 'bits' are supplied to the input ports, and the signal obtained at the output port that will be used to determine how much of a desired logical function the material encodes. (Color figure online)

of the diameter of a particle. This is at present an arbitrary design choice. The frequency of the applied signal (ω) is also fixed to 7, chosen based on the frequency spectrum of a typical random configuration of light and heavy particles, which will be discussed in the next section. The truth table in Fig. 1 shows our bit representation. The output signal $O_{ij}(i, j \in 0, 1)$ denotes the amplitude of the displacement of the particle at the output port. The three particles chosen to represent the three ports is also currently an arbitrary design choice.

Using zero and non-zero amplitudes directly to interpret bits arriving at the output port however is problematic: excitation of both input ports biases the system to produce larger amplitudes at the output port compared to excitation of just one input port, biasing any material configuration toward linear functions. Thus we instead use low and high gain at the output port to represent bits. In each of the four input cases, the gain of the system is defined as the amplitude of the fast Fourier transform ($\hat{f}$) at the driving frequency (ω) at the output divided by the sum of the amplitudes of the fast Fourier transform at the driving frequency (ω) in the inputs:

$$G_{ij} = \frac{\hat{f}(O_{ij})}{\hat{f}(\mathrm{in}_i) + \hat{f}(\mathrm{in}_j)} \quad i, j \in 0, 1 \tag{1}$$

In our experiments, for each material we measure the gain for each of the four input cases. In order for the material to act as a logic gate, the relative magnitude of the gain in each case must be consistent with desired functionality of the gate. For example, for an AND gate when both input ports are driven with a sinusoidal wave, we expect to see a high amplitude of oscillation at the output and therefore we expect a high gain (G_{11}). But in the other three cases

(00, 01, 10) we expect a low gain. Based on this, we can measure the similarity of any material's functionality to a desired logic gate by taking the distance between the four expected output bits and the four gain values.

3 Simulation Setup

In this section, we first present some details of our 2D granular metamaterial simulator. Then, we provide details of the evolutionary algorithm used for optimization.

3.1 2D Granular Simulator

We model a simplified granular metamaterial inspired by [19]. The system is 2D and composed of frictionless circular disks with fixed and equal diameters. The particles can be assigned differing masses and/or stiffnesses. The particles are placed on a 5 by 6 hexagonal lattice, resulting in 30 particles available for optimization. The system has a periodic boundary condition in the x direction and a fixed boundary condition in the y direction. There is no gravity in the system and the only forces acting on the particles are the result of a purely repulsive linear spring potential between the disks which can be formalized as Lennard-Jones potential. This system is simulated using Discrete Element Method (DEM). At each simulation time step, repulsive forces are calculated for those particles in contact with other particles, based on their distances to particles with which they are in contact. Then the accelerations, velocities and positions of each particle are updated using Verlet integration. Before probing the bulk properties of the material, a post-processing step is taken to ensure that the system is at equilibrium: the sum of the total forces between particles is near zero. This ensures that the particle packing is statistically stable. This is done by calculating the total force acting on each particle and updating their positions using the steepest-descent method to reduce total force. In the experiments where we have particles with different stiffnesses, we need to find the stable initial positions for each configuration separately. This will increase the total simulation time of our optimizations. In those cases Fast Inertial Relaxation Engine (FIRE) was used in order to reduce computational effort.

As computational metamaterials must selectively amplify or extinguish certain input waves to perform logical functions in the frequency domain, it is useful to take a closer look at their frequency spectrum. One useful property of granular metamaterials is the existence of band gaps in their vibrational density of states [2]: a contiguous range of input frequencies extinguished by the material. To locate a material's band gap, its mass-weighted dynamical matrix is calculated using the Hessian of the total potential energy. The eigenvalues of this matrix are the eigenfrequencies of the system and the eigenvectors are the modes. If the eigenfrequency spectrum is plotted by sorting the frequencies in increasing order, gaps in the spectrum become visible. The widest gap is denoted as the *band gap* (An example is shown in Fig. 2c). If the granular system is excited at a frequency within the band gap, the signal will not propagate through the material. On the other hand if the signal is outside the band gap, the system will be excited at one of its resonant frequencies and the output will be magnified. For this reason, we will choose input frequencies near the low and high cut-off frequencies of a typical material's band gap to facilitate the evolution of computational metamaterials capable of selectively amplifying or suppressing input waves, as explained in the next section.

3.2 Optimization Method

For optimization we use Age-Fitness Pareto Optimization (AFPO) [14]. AFPO is a multi-objective, multi-deme evolutionary algorithm that periodically injects new random individuals into the population and temporarily reduces selection pressure on their resulting lineages, thereby achieving diversity maintenance without requiring additional hyperparameter tuning. In all experiments we employed a direct encoding scheme for the genome: length-30 binary vectors indicated which particles were light or heavy in the mass-contrasting experiments, and which particles were soft or stiff in the stiffness-contrasting experiments. Two way tournament selection was employed to select which individuals produced offspring. Offspring were mutated by flipping each bit with probability 0.05. Crossover was not employed because there is no evidence that combining parts of two materials preserves any of their respective bulk behaviors. In all the experiments (unless mentioned otherwise) a population size of 50 was used, and each evolutionary trial was conducted for 200 generations. Three replicates were performed for each experiment. Each replicate began with a different random initial population. The fitness function for each experiment will be introduced in the subsequent sections.

4 Results and Discussion

In the first experiment, the goal was to evolve a particle configuration that maximizes the band gap. This evolved band gap was used to choose input frequencies for the subsequent experiments in which AND and XOR gates are evolutionarily embedded into two different metamaterials: those with mass-differing particles and those with stiffness-differing particles. Source code for all of the experiments is available in our GitHub repository.

4.1 Evolution of an Acoustic Band Gap

Vibrational frequency band gaps can be used to shield materials from vibrations and other perturbations. As mentioned before, vibrations with frequencies within the band gap do not propagate into the material. To block vibrations over a range of frequencies, wider band gaps are necessary. We can also tune the location of the central region of the band gap to block perturbations over different frequency ranges. A granular metamaterial's band gap can be altered by changing the number of particles embedded within it, as well as the particles' masses, positions, stiffnesses and shapes. Here, we focus on particle arrangements and define the optimization problem as finding the placement of a fixed number of heavy and light particles on a hexagonal lattice to maximize the band gap. In this section, we assume that we have 9 light and 21 heavy particles (30 particles in total), an arbitrary design chose at present. The genome encodes the positions of the light particles on the lattice. The result of three evolutionary trials is shown in Fig. 2.

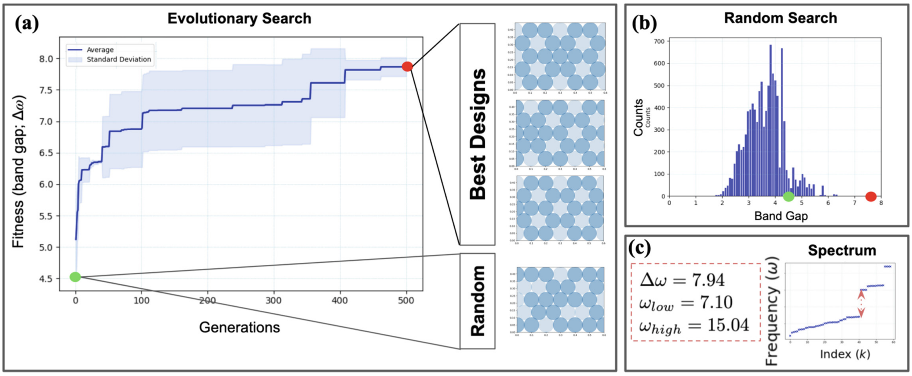


Fig. 2. Evolving the placement of 9 light and 21 heavy particles on a hexagonal lattice to maximize the acoustic band gap. (a): The fitness curve (solid blue) reports evolutionary progress as the fitness of the best individual from each generation averaged over 3 runs. The three most fit solutions from three independent runs are shown on the right. (b): The histogram shows the distribution of 10000 randomly generated samples. (c): The plot shows the frequency spectrum of the best solutions along with the width, start and end points of the band gap on the left. (Color figure online)

This problem admits $C_9(30) = \binom{30}{9} = 143071150$ possible materials. To judge the quality of optimization, we generated 10000 random configurations and calculated the band gap for each of them (Fig. 2b). The mean value of this distribution lies near 4. At the end of 500 generations (with a population size of 30), AFPO was able to find configurations with a band gap of $\Delta\omega = 7.94$. It's worth mentioning that in this section, we chose to increase the number of generations, because probing the evolutionary progress showed a continued improvement. Interestingly, the optimal designs with the highest band gaps are symmetric and show an ordered arrangement, which is consistent with our knowledge from materials sciences. Band gaps are known to occur in crystalline mixtures with regular patterns [3]. Moreover, because of the periodic boundary condition in the x direction, the three best solutions are the same configuration, just shifted different distances horizontally.

4.2 Evolving an AND Gate

In this experiment, particle configurations are evolved on a hexagonal lattice to act as much like an acoustic AND gate as possible. As we mentioned in the simulation section, we can measure the gain of the system (the relative amplitude of output oscillations to the input oscillations) for each of the four possible input cases $(00, 01, 10, 11)$. G_{00} is trivial: if the input is 00—no displacement is applied to either of the input particles—the output particle will yield no displacement either. For the other three cases, a significant gain should only be observed when both inputs are activated (high G_{11}). To achieve this, we defined the following fitness function:

$$f_{\text{“AND-ness”}} = \frac{G_{11}}{(G_{10} + G_{01})/2} \tag{2}$$

The next two sections present the results of evolving mass-varying and stiffness-varying granular metamaterials with this fitness function.

4.2.1 Evolving an AND Gate in a Mass-Varying Material

Figure 3 reports the results of evolving the placement of light and heavy particles, with a mass ratio of 10, using Eq. 2 as the fitness function.

The histogram in panel (a) shows the distribution of 5000 random configurations sampled from $2^{30} = 1.07 \times 10^9$ total possible configurations based on the measure of “AND-ness” (Eq. 2). The mean AND-ness for a random configuration is 0.715. The best configuration found by random search has a fitness of 3.52. Optimization was able to find a configuration with a fitness of 8.21 after 200 generations. Figure 5 reports the fittest designs from the three independent evolutionary trials. We notice that these configurations are not intuitive or symmetric, which makes it much harder to obtain from scratch in material design. Figure 4 illustrates how one of these best designs approximates the behavior of an AND gate: there is significant gain in the signal at the output port, at the driving frequency, only when both input ports are excited at that frequency.

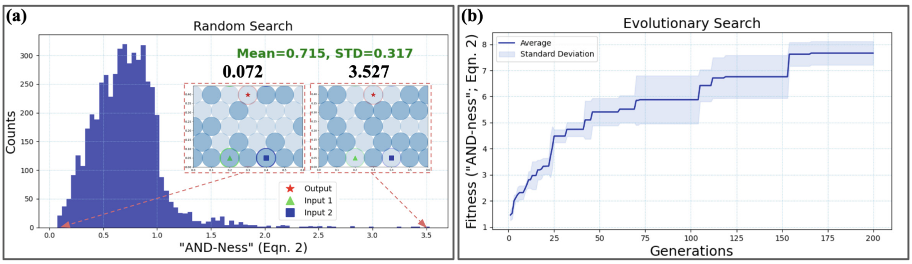


Fig. 3. Designing an AND gate in a mass-contrasting assembly of particles. (a): The histogram shows the distribution of the AND-ness in 5000 random configurations. (b): The plot shows the progress of optimization during 200 generations.

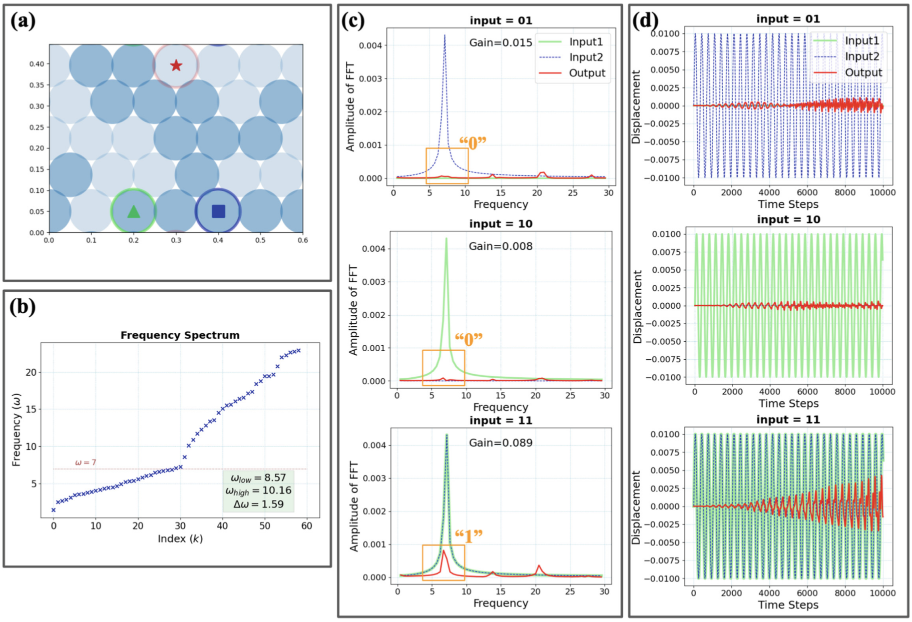


Fig. 4. (a): One of the best designs for an AND gate with mass-contrasting particles. (b): its band gap characteristics. (c): amplitudes at the driving frequency (7), and all other frequencies, at the input and output ports, for three of the four input cases, in the frequency domain. (d): the same signals, shown in the time domain. The orange rectangle in each of the plots in panel (c) highlights the behavior of the output port. The 00 → 0 case is not shown as it is trivial and always holds, regardless of material, because no energy can enter the material except through the input ports. (Color figure online)

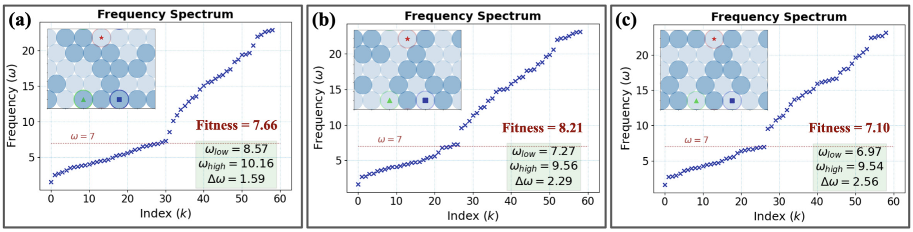


Fig. 5. The fittest AND gate designs from the three evolutionary trials, using mass-contrasting particles. Their frequency spectra, band gap features, and fitness values (Eq. 2) are also shown.

4.2.2 Evolving an AND Gate in a Stiffness-Varying Material

To investigate how different materials facilitate or obstruct the ability to evolve computational abilities into them, we evolved materials composed of particles with the same mass but differing stiffnesses: AFPO places stiff and soft particles, with a stiffness ratio of 10, and evolves materials to maximize “AND-ness” using Eq. 2. Figure 6 reports the result of optimization. As seen in the histogram, mean AND-ness for configurations found by random search is 0.760. The best configuration found by random search has a fitness of 7.88. AFPO performed significantly better than random search: in one of the three trials, it found a configuration with a fitness of 10.61 after 200 generations. Figure 7 shows how this configurations acts as an AND gate. Figure 8 shows the three best designs from the three trials.

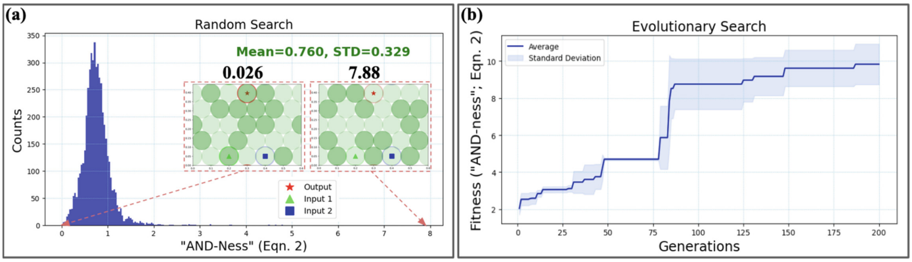


Fig. 6. Designing an AND gate in a stiffness-contrasting assembly of particles. (a): The histogram shows the distribution of “AND-ness” in 5000 random configurations, as well as the best and worst material found. Light and dark green colors indicate soft and stiff particles, respectively. (b): The plot shows the progress of optimization during 200 generations. (Color figure online)

4.3 Evolving an XOR Gate

An AND gate is not that far from a linear function, as more energy put into the system at the two input port should produce more energy at the output port, at least when both inputs are activated. Thus we next attempted to evolve an XOR gate into materials, as it is a more non-linear function and thus would intuitively seem to require more design effort. In an XOR gate, we expect to see a significant displacement at the output if only one of the input ports is being driven by a sinusoidal displacement (the 01 and 10 input cases). In order to achieve this, we defined an “XOR-ness” fitness function as follows:

$$f_{\text{“XOR-ness”}} = \frac{(G_{10} + G_{01})/2}{G_{11}} \tag{3}$$

such that increasing values denote materials that act increasingly like an XOR gate. As before, we investigated evolving materials with mass-contrasting particles and materials with stiffness-contrasting particles against this fitness function.

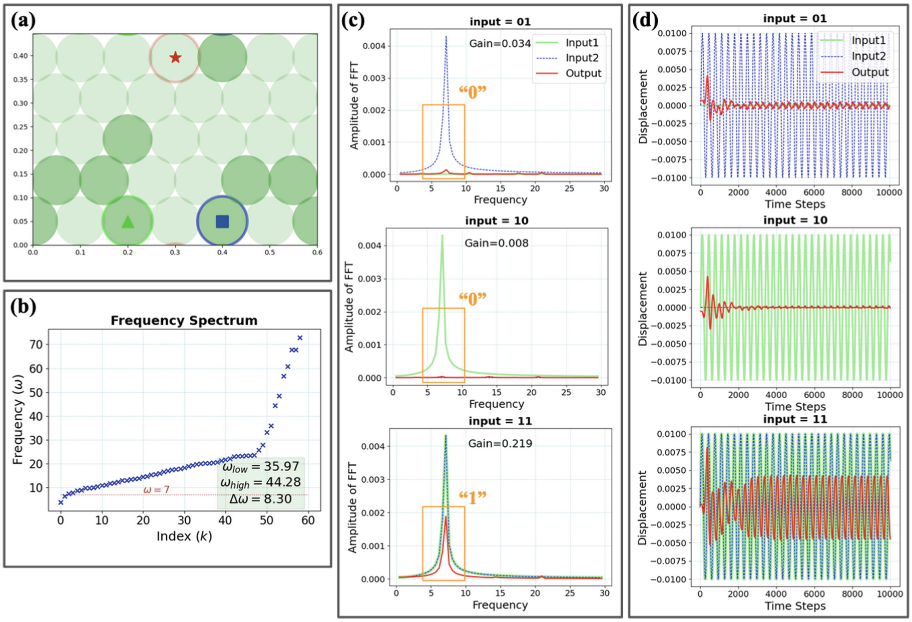


Fig. 7. One of the best designs for an AND gate with stiffness-contrasting particles. (a) and (b) show the configuration and its band gap respectively. (c) shows the response of the material at the driving frequency, and the other frequencies, for three of the four input cases. (d) shows the displacements of the input ports and the output port over time.

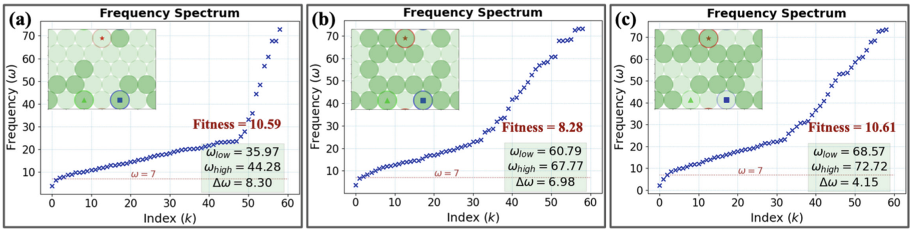


Fig. 8. The most "AND-like" stiffness-varying materials from the three evolutionary trials. The band gap for each configuration is shown below each of them.

4.3.1 Evolving an XOR Gate in a Mass-Varying Material

We performed three evolutionary trials that optimize the placement of heavy and light particles (mass ratio = 10) into materials such that they maximize "XOR-ness" (Eq. 3). Figure 9 shows the result of optimization. There we see that mean XOR-ness in materials found by random search is 1.775, while the best material had an XOR-ness of 26.59. Evolutionary search performed significantly better: it

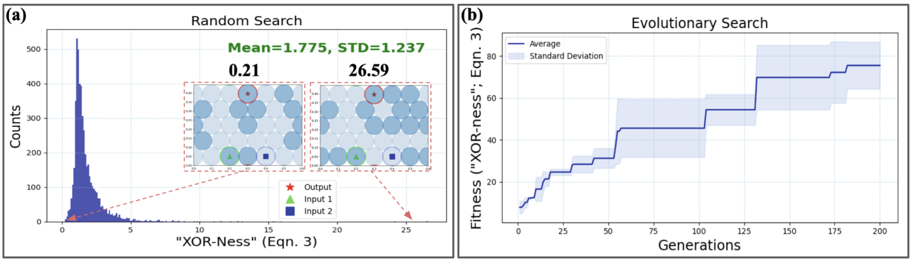


Fig. 9. Designing an XOR gate in a mass-contrasting assembly of particles. (a): The histogram shows the distribution of XOR-ness across 5000 materials found via random search. (b): Progress of evolutionary optimization.

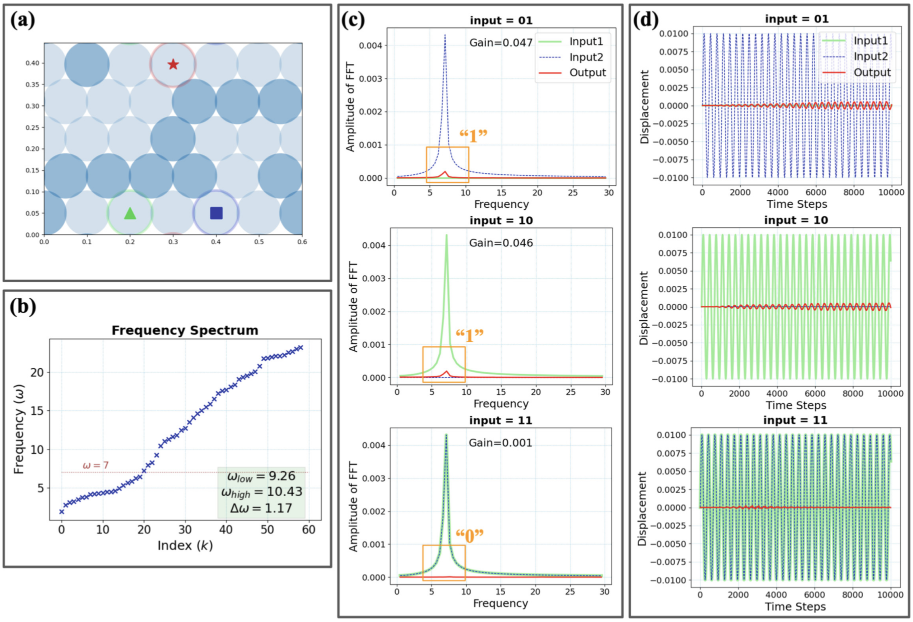


Fig. 10. One of the best designs for an XOR gate found for materials with mass-contrasting particles. (a) and (b) report the evolved configuration and its band gap features. (c) reports the material's response at the driving frequency (7) and all other frequencies. (d) shows material's behavior over time when presented with three of the four input cases.

found a material with an XOR-ness of 91.08. Figure 11 shows that material and the best materials found in the other two trials. Figure 10 reports the detailed behavior of one of these materials.

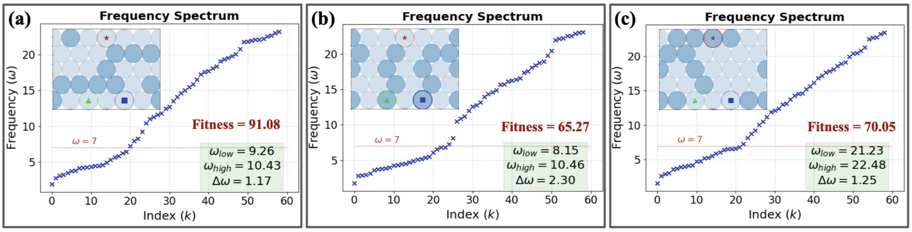


Fig. 11. The most XOR-like mass-varying materials found by each of the three evolutionary trials. The frequency spectrum of each configuration along with its fitness values is shown below each one.

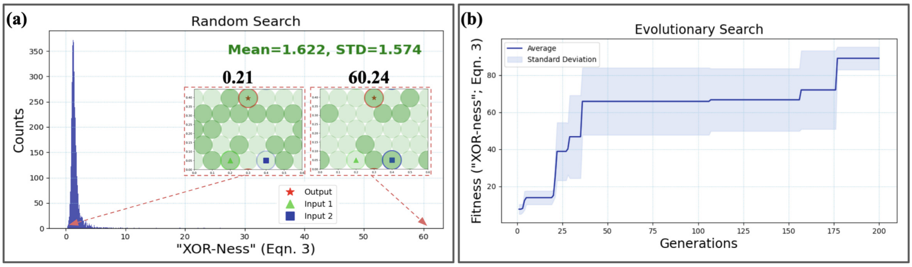


Fig. 12. Evolving XOR behavior into a stiffness-contrasting assembly of particles. (a): The distribution of XOR-ness (as defined in Eq. 3) in 5000 materials found by random search. (b): Average performance of the three evolutionary trials.

4.3.2 Evolving an XOR Gate in a Stiffness-Varying Material

As with the AND gate experiments, we investigated whether different kinds of materials facilitate or obstruct evolving XOR gates into them. To do so, we performed another three evolutionary trials using the XOR-ness fitness function (Eq. 3) on materials in which 30 soft or stiff particles could be placed into the material. Figure 12 reports the relative performance of random and evolutionary search. Although the mean fitness of materials found by random search is only 1.622, the best material achieved an XOR-ness of 60.24. Evolutionary search, on the other hand, again performed significantly better than random search: it found a particle assembly with a fitness of 94.08. Figure 13 shows one of the three best solutions from evolutionary optimization. Figure 13c shows that this material does indeed act as an XOR gate: the amplitude of oscillations at the output is significantly higher when only one of the input ports is activated. Figure 14 shows the best solutions from the three trials.

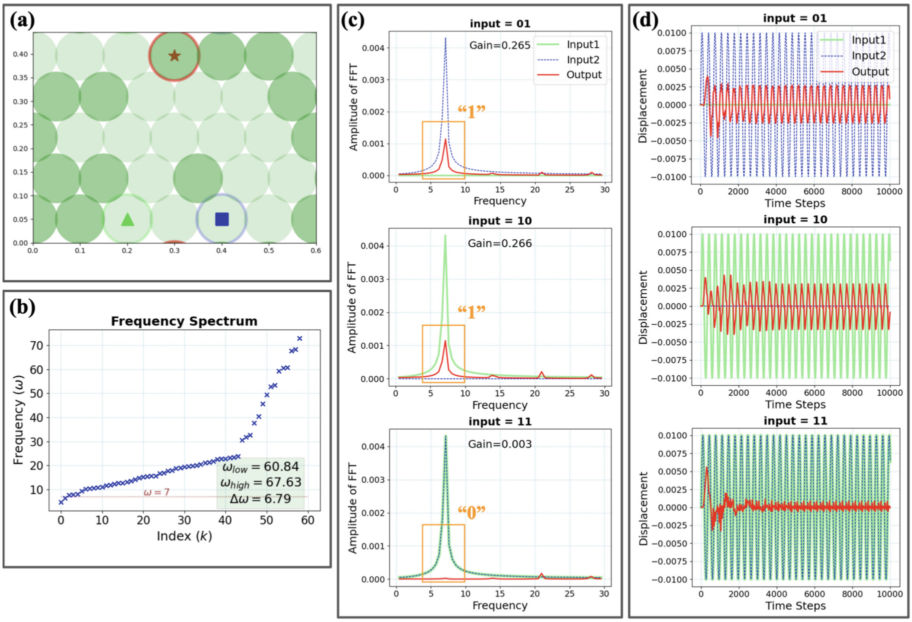


Fig. 13. One of the most XOR-like evolved materials, composed from stiffness-contrasting particles. (a) and (b) report the configuration and its band gap features. (c) reports how the material responds at the driving frequency (7) and all other frequencies. (d) shows how the three ports displace, over time, during three of the four input cases.

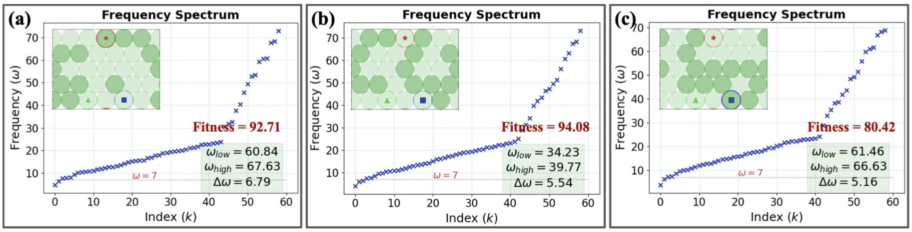


Fig. 14. The most XOR-like stiffness-varying materials, produced by each of the three evolutionary trials. The frequency spectrum of each configuration along with its fitness is shown below each one.

5 Conclusion and Future Work

Here we demonstrated the potential for granular metamaterials to act as physical substrates for computation expressed as amplification or extinction of acoustic waves. The significant performance advantage obtained by evolutionary search over random search, for two different logical operations and two different kinds of granular materials, indicates that such materials can embed computation, but finding ones that do is non-trivial, even for simple Boolean operations. Moreover, as the computation becomes more challenging, the efficacy of evolutionary over random search increases: evolutionary search is 10 times better at finding materials that act like AND gates compared to random search (Figs. 3, 6); it is 100 times better at finding materials that act like XOR gates (Figs. 9, 12).

Moreover, we noticed that evolutionary search can embed computation into some materials better than others. For example, the most AND-like mass-varying material shows a much weaker '1' signal at its output port (Fig. 4c) compared to the '1' output by the most AND-like stiffness varying material (Fig. 7c). Similarly, the '1's output by the most XOR-like mass-varying material (Fig. 10c) are weaker than the '1's output by the most XOR-like stiffness-varying material (Fig. 13c).

The non-intuitive nature of embedding computation into granular metamaterial is also evidenced by the lack of obvious common patterns across the evolved materials that best embody the logic gates: each has unique ratios of light/heavy or soft/stiff particles, geometric patterns, and there is no obvious regularity or symmetry (Figs. 5, 8, 11 and 14). This emphasizes the utility of automated design in this domain. Designing a configuration of particles to behave as a logic gate is a rather difficult task to accomplish without the aid of computer optimization.

Future work is planned in which the design space is expanded by expanding lattice resolution, and subjugating particle shape and input/output port placement to evolutionary optimization as well. Analytic efforts will focus on attempting to understand how vibrating particles encode computation by training machine learning methods to seek common patterns across successful designs not visible to human inspection. We will also investigate whether successful designs compute sub-functions in different regions of the material and then combine them downstream, or do not need recourse to such divide-and-conquer strategies.

We will also explore verifying our simulation results in physical hardware. It is possible that, given the discrete nature of granular metamaterials compared to continuous media, crossing the reality gap may prove easier for former compared with the latter. Also, because different designs are currently just different placements of two types of particles on a predefined grid, we expect the fabrication process to be cheaper, faster and easier as well.

Granular metamaterials, unlike continuous media, afford the possibility of serving not just as computational substrates but as reconfigurable computational substrates: it may be possible to build physical GMMs from particles that dynamically change stiffness in response to external stimuli such as temperature. This may allow for the packing of more computational ability within the same dynamic material. Creating increasingly computationally dense GMMs will also be investigated by providing waves with increasingly complex and diverse shape, at more input ports, with summed oscillatory components that drive different computations in the same material at the same time. This may in time show that granular metamaterials, or other emerging exotic materials, may be competitive with or possibly superior to current electronic devices as vehicles for computation.

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